Accepted Manuscript

A Mechanistic-Empirical Method for the Characterisation of Railway Track Formation

Horacio Alejandro Mones Ruiz Martin, Petrus Johannes Gräbe, James William Maina

PII: S2214-3912(18)30163-6
DOI: https://doi.org/10.1016/j.trgeo.2018.10.003
Reference: TRGEO 204

To appear in: Transportation Geotechnics

Received Date: 24 July 2018
Revised Date: 3 October 2018
Accepted Date: 11 October 2018

Please cite this article as: H.A.M. Ruiz Martin, P.J. Gräbe, J.W. Maina, A Mechanistic-Empirical Method for the Characterisation of Railway Track Formation, Transportation Geotechnics (2018), doi: https://doi.org/10.1016/j.trgeo.2018.10.003

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
A Mechanistic-Empirical Method for the Characterisation of Railway Track Formation

Horacio Alejandro Mones Ruiz Martin¹
¹Department of Civil Engineering, University of Pretoria, Engineering I Office 12-10, Lynnwood Road, Hatfield, Pretoria 0002, South Africa
Email: horacio.monesruiz@up.ac.za

Petrus Johannes Gräbe¹
¹Department of Civil Engineering, University of Pretoria, Engineering I Office 13-7, Lynnwood Road, Hatfield, Pretoria 0002, South Africa
Email: hannes.grabe@up.ac.za

James William Maina¹
¹Department of Civil Engineering, University of Pretoria, Engineering I Office 13-5, Lynnwood Road, Hatfield, Pretoria 0002, South Africa
Email: james.maina@up.ac.za

ABSTRACT: The measurement of track substructure deflections is an important parameter for the determination of structural capacity, identification of track problem areas and evaluation of overall track condition. This paper describes a mechanistic-empirical method by which elastic moduli of railway formation layers can be determined from substructure deflections under transient train loading. The method was founded on surface deflection theory from falling weight deflectometer (FWD) analysis. Finite element (mechanistic) analysis and field data (empirical) from railway substructure deflections obtained from multi-depth deflectometers (MDDs) were used to assess the validity of the mechanistic-empirical method by comparing measured and modelled railway substructure responses. The results indicate that substructure deflections and stresses are affected by the complex superposition of different bogie loading configurations on a particular superstructure. The load distribution in the railway substructure does not follow a 45° influence line as commonly assumed in surface deflection theory. On the contrary, railway equilibrium influence lines are significantly influenced by the structural capacity of the formation layers and in-situ subgrade. The predictions offered by this method strongly agree with the long-term formation peak strains measured by MDDs and this method was determined suitable for evaluation of formation structural capacity due to good agreement between measured and estimated elastic moduli of formation layers.

KEYWORDS: Surface deflection theory; Multi-depth deflectometer (MDD); Inverse analysis; Stress distribution lines; Deflection basins.
1 INTRODUCTION

The demand on railway freight lines is constantly growing with respect to train speed and axle loading. This demand has encouraged the railway engineering industry to strive for a better understanding of track formation layer (i.e. subballast and fill subgrade) behaviour through the development of practical and effective deflection measurement techniques. These techniques, in addition to what is provided by visual track inspections, are aimed at determining formation structural capacity, predicting long-term performance and developing cost-effective repair and maintenance strategies for longer service life of railway freight lines.

Several deflection measurement techniques have been developed for evaluating the structural capacity of railway track formation under static or dynamic impulsive loading through estimation of elastic moduli and/or stiffness values of the substructure layers (Gräbe et al., 2005; Sussmann and Selig, 2000). The most common techniques are depth deflection profiles and surface deflection basins, which are measured by multi-depth deflectometers (MDDs) and the falling weight deflectometer (FWD) respectively (De Beer et al., 1989; Horak, 2008; Jooste, 1992). With the availability of deflection data, a need was identified to develop a mechanistic-empirical method that can accurately evaluate the structural capacity of the track formation through analysis of the full deflection basin with train passage. Furthermore, the ideal method should be inexpensive, non-destructive, simple to install and operate, allow measurement at different locations, provide high levels of deflection measurement reproducibility and estimate structural integrity under train transient loading.

This paper therefore provides insight into the development of an innovative mechanistic-empirical method based on surface deflection theory that will allow estimation of formation elastic moduli under train transient loading using an inversion technique.

2 RAILWAY FORMATION BEHAVIOUR

As background to this paper, this section deals with theoretical aspects of railway track loading and deformation with a focus on the railway formation layer (i.e. subballast and fill subgrade) behaviour under train transient loading.

2.1 Loading Condition

The main objective of formation materials is to sustain the train loading by spreading the induced stresses to the underneath layers over a larger area to ensure that substructure layers are not overstressed at any given time during their service life (Li et al., 2016). This paper focuses on train transient loading which is a combination of cyclic and dynamic loading.
Cyclic loading is repeated train wheel loading pulses that dissipate with depth in relation to the superstructure components and the substructure condition. The effect of cyclic loading on railway formation is influenced by the following factors: load shape, load duration, loading pulse magnitude, time interval between successive loading cycles and total number of loading cycles (Li et al., 2016). In terms of train transient loading, these influential factors can be translated into train operational speed, axle loading, bogie wheel spacing, train length and total experienced traffic. Furthermore, the cyclic loading rate (train travelling speed) and its variation (axle loading) has an effect on the strength and stiffness of the supporting layers. This effect is relatively small for ballast and subballast materials, while subgrade materials tend to be more susceptible to the loading rate. All granular materials are therefore susceptible to cyclic loading effects, especially under saturated, undrained conditions (Li, 1991).

Dynamic loading is generally characterized as short or long duration forces. Short duration forces are high frequency impact loads resulting from localized structural imperfections, such as rail or wheel discontinuities. Long duration forces are low frequency dynamic loads produced by the long wave lengths due to track geometry irregularities (Li et al., 2016). Dynamic wheel loads can also occur as a result of bogie miss-alignment and variation in rail temperature inducing transverse and longitudinal forces respectively (Jenkins et al., 1974). A dynamic increase in cyclic formation loading commonly causes localised formation distress due to overstressing of the various track components which can lead to ballast breakage and an overall reduction in sleeper support at a specific location (Gräbe et al., 2005). This decrease in sleeper support generates an increase in formation loading which influences the contact forces, stress path and levels. This results in higher stresses deeper in the substructure, eventually weakening the material and causing a structural impairment that leads to severe formation deformation (Gräbe et al., 2005; Lundqvist & Dahlberg, 2005; Priest & Powrie, 2009; Wilk et al., 2016; Yang et al., 2009).

Resilient deformation has the ability to define the performance of the track (Selig & Li, 1994), based on the notion that the resilient component of granular soils is not a constant but rather varies according to the loading condition, soil characteristics and physical soil state (Lekarp et al., 2000a; Li & Selig 1996; Li et al., 2016; Soliman & Shalaby, 2015). In summary, the rate of resilient deformation is dependent on the material strength and behaviour under repetitive loads.

3 DEFLECTION MEASUREMENTS

This section discusses the interpretation of deflection measurements with a focus on surface deflection theory for pavement condition evaluation, characterising the curvature of the deflection basin as well as the importance of influence lines and factors affecting the shape of deflection basins for this investigation. The literature presented here is mostly based on road pavement research, given that railway and pavement substructures behave in similar manners. In this regard, the literature serves as a background to the principles behind surface deflection theory for further understanding and development of the method presented in this paper.
3.1 Curvature of Deflection Basins

The analysis of pavement deflection measurements has been widely researched in South Africa by De Beer et al. (1989), Horak (2008) and Jooste (1992), utilising FWD as well as MDDs for pavement structural evaluation. The analysis of deflection basins has been used to identify weak areas in the depth of the pavement structure and over the length of a uniform section without the need for in-depth investigation by simply analysing the shape of deflection measurements (SAPEM, 2014).

The deflection of a pavement structure is the response to induced loads which generates stresses in the system that dissipate with increasing depth. Thus deflection basins tend to vary in size and shape depending on the following factors: pavement composition, structural strength, load contact area, load magnitude, loading durations, instrumentation utilised and climate changes. Therefore, it is imperative to describe pavement behaviour based on the entire deflection basin curvature, rather than through limited deflection points (Horak, 1988; Maina et al., 2009).

Previous publications by Horak (2008) and Li et al. (2016) further support the notion that different zones of the deflection basin represent different structural layers, showing that the shape of pavement deflection basins under wheel loading can generally be classified into three distinct zones of curvature as illustrated in Figure 1. The positive curvature (closest to the point of loading), indicating the structural condition of the upper layers; the inflected zone indicating the condition of the middle layers, being highly influenced by the pavement structural composition; and lastly the reverse curvature (furthest from the point of loading), representing the structural response of the lower layers, being governed by the actual depth of the pavement structure.

3.2 Influence Lines on Deflection Basins

The stress distribution, also known as the influence line or influence zone, is affected by the pavement loading characteristics and the individual material properties in the pavement system namely elastic modulus \(E\), layer thickness \(t\) and Poisson’s ratio \(v\) (Jooste, 1992). Thus, the deflection of a multi-layered system is proportionally governed by the stress distribution angle of the individual layers as investigated by Zakeri and Mosayebi (2016) in ballasted track structures. However, the pavement wheel loads are normally assumed to spread from the top layer to the underlying layers through an influence line of about 45° (Horak, 2008), where any material above this line is assumed not to contribute towards supporting the applied load.

Figure 1 shows the zone of influence and the proportional contribution of a multi-layered pavement system (Ullidt, 1987). The maximum total deflection \(\delta_{t}\) is measured beneath the applied load and it decreases as the applied load dissipates into the layered pavement system. The estimated deflections of the layered system are calculated corresponding to the intersection of the layers with the assumed influence line. For instance, the proportional
The deflection of the base layer is estimated by deducting the measured deflection of the base/subbase layers’ ($\delta_2$) intersection from the maximum total deflection ($\delta_t$).

Furthermore, as the horizontal distance increases, a point is reached where only the subgrade ($L_4$) falls within the zone of influence, reflecting the subgrade deflection only. If this method is then inversely applied, working from the bottom layer towards the uppermost layer, it is possible to obtain the proportional deflection of each individual pavement layer (Horak, 1988, 2008; SAPEM, 2014). Subsequently, obtaining the relative deflection of each contributing pavement layer, it is possible to iteratively and inversely estimate the pavement moduli from a single surface deflection basin and indicate the structural capacity of the different layers (De Beer et al., 1989; Horak, 2008; Jooste, 2002).

Pavement analysis is commonly conducted using a half-space approach for the deflection basin of the leading wheel load, due to potential plastic deformations and the probability of superimposing wheel load effects from trailing wheels at different speeds, as shown in Figure 1. For that reason, Horak (1988) advised manipulating the deflection measurements so that the maximum deflection is set as the origin of the deflection basin and the half-space deflection curvature remains unchanged until zero deflection is measured, ensuring that the data is relatively free from plastic deformation and errors in the deflection basin curvature. This has been widely used in FWD analyses.

Figure 1: $45^\circ$ load distribution through pavement layers (adapted from Ullidtz, 1987)
3.3 Factors Affecting the Shape of Deflection Measurements

According to previous research by Horak (1988) and Maina et al. (2009) on pavement structures, factors that may affect the formation deflection shape (i.e. the shape of the deflection vs. time plot) under train transient loading can be hypothesised as superstructure characteristics, formation structural capacity, loading configuration, instrumentation utilised and climate change.

The influence of substructure components on formation deflection, similar to road pavements, is associated with the structural capacity and thickness of each individual formation layer as well as the in-situ subgrade support (stiffness) which affect the magnitude and shape of the measured formation deflections (SAPEM, 2014). Tam (1985) also investigated the effect of granular base and subgrade stiffness on deflection basins concluding that the variation of base layer thickness had the greatest influence on the maximum deflection and spread of the deflection basin, followed by the subgrade and base layer stiffness. The results further indicated that the weaker the base or subgrade stiffness, the higher the maximum deflection and spread. In addition, Ullidtz (1987) indicated that the subgrade strength contributes between 60% and 80% of the maximum peak deflection.

Railway track formation has similar responses to the variation in layer thickness and stiffness as those of road pavement structures. However, ballasted railway track structures have complex stress distributions due to the indirect load transfer through the superstructure (Selig and Waters, 1994) which is different from that of road pavement structures where the loads are directly transferred to the pavement structure. It is therefore essential to investigate the superstructure effects on formation stresses and deflections. According to Lundqvist and Dahlberg (2005), the formation deflection is affected by superstructure aspects such as sleeper spacing and support. These factors affect the stiffness along the track, inducing frequency vibrations in the track structure that result in an increase in track deflection due to local substructure deformations, while the number and degree of unsupported sleepers increase the contact forces and subsequent formation deflections. Zakeri and Mosayebi (2016) also indicated that the reduction in sleeper spacing and the increase in ballast depth and fouling content are factors that can particularly influence the load transfer to the formation resulting in a stress path overlap in the ballast layer, subsequently increasing the stresses and deflections of the formation layers.

Track formation deflection is further influenced by the complex interaction between loading configuration and the superstructure characteristics. In brief, bogie spacing and number of axles of the locomotive/wagon do not necessarily match the sleeper spacing. Thus, the axle load transfer results in a complex superposition which can be transferred to either the nearest sleeper or, alternately, can be subdivided between the two adjacent sleepers depending on the position along the track (Li et al., 2016). Hence, formation deflection basins may vary according to the rolling stock and ballasted track characteristics. Furthermore, the effect of train transient loading on formation deflection is dependent on the rolling stock characteristics i.e. axle load, axles/bogie, bogie spacing and train speed. The effect of these characteristics on the shape of railway substructure deflection is shown by Li et al. (2016). Vorster and Gräbe
(2013) investigated the effect of axle loading on formation deflection concluding that deflection increases less rapidly with higher axle load, following a logarithmic relationship.

The increase in train speed also affects the formation deflection as it decreases the effective track modulus, consequently resulting in larger effective axles loads, formation deflection and also reducing the resolution of the deflection measurements (Priest and Powrie, 2009). Yet, Yang et al. (2009) indicated that these effects are mainly observed if the train speed approaches the critical speed of the track system, therefore not applicable to the South African Heavy Haul Coal Export Line used in this study where trains do not exceed the critical speed. In the case of trains not approaching the critical speed, it is generally seen that the maximum vertical formation deflection occurs directly beneath the leading train axle, reducing with depth and altering from axle loading to bogie loading with increasing depth (Priest et al., 2010).

The most important climatic factors influencing pavement structures are temperature, moisture changes and the effect of frost (Ullidtz, 1987). The temperature variation can be defined as an insignificant factor for railway track structures since the substructure is mainly constituted of unbound granular materials which are not susceptible to temperature differences (SAPEM, 2014). However, there is evidence that granular materials can expand with temperature variation causing a “lock up” behaviour which stiffens the material and leads to lower deflections (Jooste, 1992). In most parts of South Africa, rainfall is the determining factor that governs the railway track performance due to the variation in moisture content. Changes in moisture content of unbound granular materials are known to affect the pavement moduli (Ullidtz, 1987) as it reduces the structural capacity and results in high formation deflections (SAPEM, 2014). This leads to the conclusion that only regions with high rainfall will be susceptible to climatic changes. Furthermore, frost rarely occurs in South Africa, therefore its effect is not discussed in order to maintain brevity.

To develop the new mechanistic-empirical method that would allow estimation of elastic moduli of formation layers under train transient loading using an inversion technique, a test site on a South African railway line was chosen where the method could be implemented and verified through the use of pre-installed instrumentation. This site is described in the next section.

4 SITE DESCRIPTION

The Coal Export Line in South Africa was constructed in 1976, connecting approximately 40 mines in the Mpumalanga coalfields with the Richards Bay Coal Terminal. During 1994-1995 signs of formation failure were observed, leading to immediate rehabilitation in 1995 until present day. The rehabilitation provided the perfect opportunity to install extensive instrumentation to monitor and characterise formation behaviour under transient train loading from the initial construction of the formation over an extended period of time. The instrumented test site at Bloubank was then constructed on the Transnet Freight Rail (TFR) heavy haul Coal Export Line between Vryheid and Richards Bay, situated 60 km South of Vryheid. The formation at Bloubank was fully rehabilitated and reopened to
traffic on 2 April 2004. The Bloubank test site has been described in previous publications by Gräbe et al. (2005), Gräbe & Shaw (2010) and Priest et al. (2010). However, the formation design and instrumentation layout are further described for understanding the terminology used in this paper.

### 4.1 Formation Design

The rehabilitated railway formation at Bloubank was built in accordance with the Specification for Railway Earthworks (S410) by Transnet Freight Rail (2006). A schematic of the design is shown in Figure 2.

The formation design entailed an 800 mm by 4000 mm excavation of old foundation material, reconstruction of the formation in four high-quality structural layers of 200 mm each at a slope of 1:25 for surface water drainage. The design also incorporated an extensive fin drain system on both sides with a geosynthetic at the interface between the B layer and the in-situ subgrade for groundwater cut-off and separation (Gräbe et al. 2015). The track superstructure comprised 60 kg/m CrMg rails, Fist fasteners and PY concrete sleepers (30 tonnes/axle capacity) spaced at 650 mm centre-to-centre on a 280 mm to 300 mm ballast layer (Gräbe and Shaw, 2010).

![Figure 2: Track and formation design at Bloubank test site (Gräbe and Shaw, 2010)](image)

The material properties per structural layer as shown in Figure 2, are summarised in Table 1. The minimum required specifications are stated in brackets after the actual value for the listed soil properties.
Table 1: Summary of selected formation material properties (adapted from Gräbe and Shaw, 2010)

<table>
<thead>
<tr>
<th>Property</th>
<th>SSB</th>
<th>SB</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel (%)</td>
<td>71</td>
<td>70</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>Sand (%)</td>
<td>21</td>
<td>22</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>Silt (%)</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Clay (%)</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Natural Moisture Content (%)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Liquid Limit</td>
<td>SP</td>
<td>SP</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Grading Modulus</td>
<td>2.5 [&gt;2]</td>
<td>2.4 [&gt;1.8]</td>
<td>1.9 [&gt;1.0]</td>
<td>1.9 [&gt;0.5]</td>
</tr>
<tr>
<td>AASHTO - Soil Classification</td>
<td>A-1-a (0)</td>
<td>A-1-a (0)</td>
<td>A-2-6 (0)</td>
<td>A-2-6 (0)</td>
</tr>
<tr>
<td>Unified - Soil Classification</td>
<td>GP-GM</td>
<td>GP-GM</td>
<td>SC</td>
<td>SC</td>
</tr>
</tbody>
</table>

4.2 Instrumentation

The test site comprised of three stations positioned 5200 mm apart with a total of 18 multi-depth deflectometer (MDD) modules and two wheel load sensors per station. Each station contained three vertical strings of MDDs namely left, centre and right strings. The strings comprised of six MDDs each positioned at various depths within the formation. The MDD strings were all installed directly after construction in order to monitor the track behaviour before reopening to traffic.

The plan view and cross section of a single station indicating the positioning of MDD stations and wheel load sensors relative to each other, are shown in Figure 3 and 4. As observed, the MDDs are located at the interface of each formation layer with exception of MDD 1 and MDD 6. MDD 1 could not be placed at the ballast/special subballast (SSB) layer interface due to the required anchoring space and to prevent module damage from ballast penetration, further reducing the likelihood of erratic measurements (Shaw, 2005). The MDD position relative to the top of the formation and the effective measuring thicknesses are presented in Table 2. The reader is referred to Shaw (2005) for further detail regarding the Bloubank test site including a detailed description of the MDD installation.
Figure 3: Bloubank test site layout indicating MDD and wheel load sensors
(adapted from Gräbe and Shaw, 2010)

Figure 4: Cross section of test station layout (adapted from Shaw, 2005)
Table 2: Details of the MDD system indicating position and effective measuring thickness

<table>
<thead>
<tr>
<th>Module</th>
<th>Layer</th>
<th>Position from top of formation (mm)</th>
<th>Effective Measuring Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SSB</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>SSB/SB Interface</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>SB/A Interface</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>A/B Interface</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>B/In-situ Interface</td>
<td>800</td>
<td>200</td>
</tr>
<tr>
<td>6</td>
<td>In-situ 200mm</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>Anchor</td>
<td>In-situ 1500mm</td>
<td>2500</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3 Train Loading

The Coal Export Line carries a mixture of freight traffic. The majority of the traffic that was measured at the test site consisted of 20 and 26 tonnes/axle coal and general freight wagons. The trains commonly consisted of either 100 or 200 wagons. In this paper only locomotive bogie loading is considered varying from 21 to 29 tonnes per axle.

5 FINITE ELEMENT MODELLING

To investigate various aspects related to the influence lines of a conventional railway track structure, a three-dimensional (3D) finite element model (FEM) was used. The FEM was developed based on the Bloubank test track and formation design as specified in the Manual for Track Maintenance and Specification for Railway Earthworks (S410) (Transnet Freight Rail, 2006). A 3D half-space linear elastic FEM representing a physical size of 4 667 mm x 19 500 mm x 6 800 mm was constructed using the Strand 7 software package. The model was based on TFR heavy haul Class Line (S) with a 60 kg/m (UIC 60) rail on a standard gauge of 1 065 mm, with concrete sleepers (PY) placed at a spacing of 650 mm on 300 mm depth of ballast and a maximum axle load of 26 tonnes.

The railway track model is shown in Figure 5. Symmetry about the ZY plane was used to reduce the number of elements required, thus achieving a computationally more efficient model. Translational and rotational restraints in the X, Y and Z directions are indicated with the symbols X, Y and Z and RX, RY and RZ respectively. Separate restraints were imposed on the superstructure (denoted in bold) and substructure. The model consisted of 108 000 hexahedral, 8-noded brick elements. The track and substructure components were modelled as elastic materials using solid elements. The granular material properties were estimated from the road and railway pavement guidelines recommended by Maree et al. (1981) and Li et al. (2016), respectively. The material properties of the track components and layers used in the model are shown in Table 3.
A single bogie of a typical locomotive for this railway line was modelled in this analysis, simulating the leading locomotive bogie of a full train. This was adopted under the assumption that other bogies of the same locomotive have no significant influence on the adjacent bogie. The formation response beneath the leading wheel load was the focus of this study. This was decided in order to mitigate complications arising from inherent variability in wagon loading, resilient deformation accumulation with repeated loading cycles during train passage and soil recovery throughout train transient loading.

The wheel loading was modelled via static point loads positioned on top of the rail crown and aligned with the neutral axis of the rail profile. The moving wheel loads were modelled by a series of static point loads spaced at 25 mm increments to simulate the transient nature of the locomotive, thus replicating continuous longitudinal movement. The
bogie was subsequently modelled in accordance with the specific locomotive being considered. Figure 6 shows the placement of the locomotive bogie wheel loads for a 7E or 11E locomotive.

In this study, the self-weight loading of the railway track structure was disregarded due to the relatively insignificant stress increase with depth caused by the superstructure and the shallow nature (less than 1 meter) of the formation, as suggested by Li et al. (2016). Thus, the focus was on the stresses and deflections as a result of train loading only.

![Figure 6: Load modelling of a single locomotive bogie](image)

### 6 FIELD DEFLECTION MEASUREMENTS

This section is dedicated to a sensitivity analysis on the influencing variables of field deflection measurements. The analysis consisted of characterising and normalizing all possible dynamic effects due to train transient loading, thus minimizing the inherent variability in substructure deflection measurements. The effect of travelling speed and axle loading on peak and spread deflection are subsequently discussed based on Bloubank test site data obtained from heavy haul operational trains measured in 2004.

#### 6.1 Dynamic Effects

Train loading generates soil vibrations which manifest as noise and random fluctuations within the deflection basin curvature, subsequently affecting the variability of the deflection measurements (Bowness et al., 2007). All possible dynamic effects caused by irregularities due to train transient loading and track geometry were minimized through normalization, thus reducing the inherent variability in substructure deflection measurements, subsequently proving the reliability of the mechanistic-empirical method (or inverse method) under changing influencing variables. To this end, left and right rail loads were averaged to find and reduce the influence of wheel and bogie defects as well as vehicle miss alignment. This allows for the distribution of any transverse imbalance within the bogie loading. Figure 7 (a) shows the elimination of deflection noise, enabling accurate estimation of the peak deflection of all three wheels. The mean peak deflection resulting from the bogie wheel load was then calculated and used to adjust the first half-spaced deflection basin of the leading axle of the bogie. This process was applied to rebalance any existing dynamic effects related to longitudinal train movement. A non-linear sigmoidal regression was therefore applied to amend the curvature variations in this study.
Conventional railway track is subjected to additional dynamic effects associated with the degree of sleeper support, which influences the contact forces, stress path and substructure deflection measurements as shown in previous studies by Lundqvist and Dahlberg (2005), Priest and Powrie (2009), Wilk et al. (2016) and Yang et al. (2009). Therefore, this study only considered fully supported sleepers as an approach to mitigate such effects. The literature further indicates that decreasing the sleeper support predominantly increases the hogging and sagging behaviour of deflection basins, as illustrated in Figure 7 (b). Consequently, this study disregarded any hogging behaviour by adopting the unloaded zero deflection line as a threshold which defines the zone of interest, also referred to as spread of deflection, subsequently minimizing inconsistencies in deflection measurements.
6.2 Effect of Travelling Speed and Axle Loading

The influence of travelling speed and axle load on the observed half-spaced deflection basin of the leading axle of the locomotive was investigated with particular attention to the deflection basin’s spread and peak, also referred to as the deflection parameters. The data presented was obtained from heavy haul trains travelling over the Bloubank test site in 2004, normalised for dynamic effects as previously described. The investigation into the effects of speed and axle loading considered both 7E and 11E locomotives with 21 and 29 tonnes/axle respectively. From the data analysis, it was noted that each Bloubank station responded slightly differently despite the same locomotive passing over the stations, therefore, a statistical analysis was conducted, identifying Station 1 as the most reliable station. The data from Station 1 was therefore used to determine the quantitative effect of train speed and axle load.

Figure 8 shows that axle loading clearly governs peak deflection for the 7E locomotive (21 tonnes/axle) ranging from 0.26 mm to 0.27 mm and from 0.36 mm to 0.37 mm for the 11E locomotive (29 tonnes/axle). The data indicates that the increase in peak deflection is directly proportional to the increase in axle loading by an incremental increase of
0.0125 mm of peak deflection per tonne. Furthermore, the deflection spread appears to be less sensitive to changes in axle loading as the spread ranged from 2600 mm to 3300 mm for both locomotives.

Only train speeds ranging from 20 km/h to 55 km/h were investigated due to the maximum allowable speed on the TFR heavy haul coal export line being 80 km/h. At this location, it was found that deflection parameters considered were not distinctively influenced by changes in train speed, thus confirming that dynamic effects at speeds lower than 60 km/h are negligible as described by Jenkins et al. (1974). Additional literature also indicated that dynamic effects on railway formation may only become apparent as the train speed approaches the critical speed (240 km/h), causing a resonance effect which increases the soil deflections (Gräbe and Clayton, 2009; Priest and Powrie, 2009; Priest et al., 2010; Yang et al., 2009). On the contrary, static deflection measurements are suspected to have higher peak and spread of deflection than transient deflection measurements (for train speeds < critical speeds) since granular materials behave in a viscoelastic manner which is a time-dependent response (Priest and Powrie, 2009).

![Figure 8: Influence of travelling speed and axle loading on deflection parameters](image)

7 INVERSE ANALYSIS

7.1 Inverse method Development

The inverse method was developed by modelling the formation layers and material properties of the South African 26 tonnes/axle heavy haul track structure (Transnet Freight Rail, 2006), as specified in Table 3. The inverse method
was developed based on 7E locomotive loading (21 tonnes/axle) and a single measurement point located at the intersection of the neutral axis of the rail profile and the centre line of the sleeper, measuring total substructure responses from the top of the formation (excluding superstructure and ballast layer). The established inverse method was developed and calibrated with field deflection measurements.

The inverse method comprised of determining the equilibrium influence lines (EILs) for conventional railway formation by calculating the transient wheel load position (also referred to as horizontal distances/boundaries) with a single sensor embedded in the top formation that result in deflections that equate to the total formation deflection relative to the wheel load positioning and the influence line. Subsequently, the total formation deflection can be translated into relative deflections that equate to the local formation deflections for the specified increments or layers as hypothesised by the shaded triangles in Figure 9. This was done to determine a true influence line, instead of adopting the commonly assumed 45° influence line for ballasted railway track. The following terminology is vital for understanding the remainder of this paper:

- Transient deflections are total formation deflection measured at a singular point as a function of a moving wheel load that approaches the measurement point;
- Local deflections are total formation deflection measured at numerous points with depth as a function of a singular centred load application.

![Diagram](https://via.placeholder.com/150)

*Figure 9: Wheel load positioning and influence lines on individual substructure layers*
For illustrative purposes, transient and local deflections were superimposed as shown in Figure 10. The EILs were estimated by means of correlating local and transient formation deflections with the objective of determining the wheel load distances that equate to the deflection of each individual formation layer. For this particular case, an iterative step process was performed for increasing increments of 200 mm allowing for the estimation of increasing transient distances corresponding to the 200 mm individual formation increments. Figure 10 shows this process in five steps (a - e) which are denoted as follows:

- **a.** Formation peak deflection beneath leading wheel load application at centre-of-sleeper;
- **b.** First 200 mm formation depth;
- **c.** Local formation deflection for the first 200 mm depth;
- **d.** Transient deflection equivalent to the first 200 mm formation depth;
- **e.** Corresponding transient distance (EILs boundaries) equivalent to the first 200 mm formation deflection.

This iterative step process was subsequently utilised to plot the EILs by plotting local vertical depth and the calculated transient horizontal distance/boundaries.

**Figure 10: Comparison of substructure deflection measurements**
Based on the above-mentioned iterative step process, different in-situ subgrade elastic modulus alternatives, varying from 50 MPa to 400 MPa, were analysed. This was done to investigate the effect of all possible substructure types on formation layer behaviour, from embankments (fills) to excavations (cuts).

Figure 11 (a) and (b) therefore show the influence of varying subgrade support on the transient deflection basin and the local deflection measurements directly beneath the leading axle of the locomotive respectively.

Figure 11 (a) confirmed that deflections resulting from a transient wheel load are the product of the substructure properties and manifest as changes in deflection basin curvature, spread and peak magnitude. The increase in subgrade support however shows an overall reduction in deflection with depth and distance. Local deflections further demonstrate the proportional contribution of the formation layers and subgrade, such as highlighting that the formation deflection curvature (300 mm - 1100 mm) remained unchanged for all subgrades considered.
Figure 11: (a) Transient substructure deflections converted to space domain; (b) Local substructure deflections; (c) Formation equilibrium influence lines for different subgrades
The EILs were estimated by means of correlating local and transient formation deflections with the wheel load distances that equate to the individual formation layers (SSB, SB, A and B).

Figure 11 (c) shows the formation EILs for the different subgrade stiffnesses in terms of vertical depth and horizontal distance. The increase in subgrade stiffness under identical formation layer material properties showed that transient deflections reduce at a smaller rate with distance than local deflections with depth, subsequently resulting in an increase in spread of formation EILs and relative horizontal distance per formation layer. This is associated with the transition from shallow to deep substructure behaviour which defines the formation deflections in relation to the increased subgrade stiffness. This clearly indicates that EILs are the product of the change in curvature of both measured deflections which are determined by the substructure material properties (elastic modulus, layer thickness and Poisson’s ratio).

Figure 11 (c) therefore proves that the railway superstructure’s discontinuous support and its indirectly induced stresses result in a different zone of influence than the prescribed 45° influence line for directly induced stresses.

A piecewise regression was used to approximate the formation EILs to minimize induced errors and allow for accurate inverse estimation of relative formation layer deflections, strains and elastic moduli. The formation strains were then calculated under the assumption that permanent deformations are negligible. The structural capacity of the railway formation is subsequently characterised by measuring transient deflection and modelling of the railway substructure vertical stresses, allowing the back-calculation of the formation layer strains and elastic moduli.

It is important to highlight that the objective of the inverse method is to calculate the unmeasurable substructure deflections with depth along the transient deflection basins, measured at a single point in space under real train transient loading and evaluate the actual substructure condition (stiffness) through known EILs. The method therefore overcomes the differences in behaviour observed between static and dynamic loading testing and allow measurement at a magnitude and frequency of loading relevant to in-service conditions (Brown and Selig, 1991) without the need for destructive instrumentation.

### 7.2 Validation of the Inverse Method

The validation of the inverse method was conducted using the Bloubank 2004 deflection measurements from Station 1 with the focus on the initial formation condition after rehabilitation. The model was adjusted to replicate the existing Bloubank test site conditions with 7E locomotive bogie loading (21 tonnes/axle) and initial structural capacity as presented by the generic formation properties in Table 3, with a 400 MPa in-situ subgrade. Furthermore, formation deflections were measured from a depth of 80 mm to 2500 mm, aligned with the neutral axis of the rail profile as well as with the positions of the MDD strings. Substructure deflections beyond 2500 mm were disregarded as the MDD anchor was positioned at this depth and all deflection measurements were referenced to this point. Therefore, the EILs
for the Bloubank test site commenced at 80 mm below the ballast/special subballast interface and incrementally increased with depth due to the inter-sleeper deflections and zero-referenced movement at 2500 mm.

Figure 12 (a) illustrates the calculated local deflection using the inverse model with conditions representative of the formation at Bloubank as well as the actual measured local deflection (Data 2004). As shown, the model agrees well with the measured formation deflections, however, a small difference is present within the first 200 mm of subgrade. This increase in relative deflection is associated with the excavation and re-compaction of the in-situ subgrade during construction, resulting in a less stiff material than the undisturbed subgrade below. This results in higher deflections ($\delta_{\text{Data}}$) than those estimated by the Bloubank inverse model ($\delta_{\text{Model}}$) which resembled a homogeneous, uniform and undisturbed subgrade.

Figure 12 (b) also compares the calculated transient deflection using the inverse model and the measured transient deflection basin (Data 2004). The illustrated EIL boundaries are the calculated transient horizontal distances that were established based on the iterative step process as shown in Figure 10. As observed the inverse model displays a clear agreement with respect to the peak and curvature of the deflection as well as deflection spread since the difference ($\delta_{\text{spread}}$) is within the expected ranges shown in Figure 8.

Validation of the inverse method involved comparison of the deflections estimated via the EIL boundaries (Figure 12 (b)) and the relative local deflections measured using the MDDs. In this regard, Figure 13 further compares the transient and locally measured peak strains. The estimated strains differed by no more than 23 % maximum (Layer A) and 10.5 % on average from the actual measured strain. As observed, the inverse method is valid for the back-calculation of railway formation properties, allowing characterisation of a railway track formation by means of transient formation deflections.

Furthermore, the inverse method can be applied to other railway formation designs and loading configurations. It is however important to understand that the method’s accuracy and reliability is dependent on deflection measurement variability, the available track condition information and a comprehension of the deflection basin variability as a function of the track characteristics and formation material properties.
Figure 12: (a) Comparison of the inverse model and Bloubank local substructure deflections measurements; (b) Estimated formation layer boundaries on transient substructure deflection measurements.
7.3 Inverse Method for Long-Term Formation Characterisation

The inverse method was established for long-term characterisation of railway formation materials with in-service loading over time. The evaluation of long-term formation response was conducted on all available data from 2004 to 2017, considering only Station 1 and 7E locomotives.

In brief, the long-term local deflection measurements showed that formation layers have inherently complicated responses. This is especially true when predicting each individual layer’s behaviour. However, local formation deflections increased monotonically with traffic, while in-situ subgrade remained naturally resilient to the generally low stresses experienced, thus supporting the assumption that the increase in substructure deflection is predominantly attributed to the structural deterioration of the formation layer with increase in traffic (million gross tonnes (MGT)).

The long-term inverse method was therefore established by equally decreasing the formation layer’s (SSB, SB, A and B) structural capacity (elastic moduli) from its initial design capacity in 2004 (Table 3) with the objective of assessing the formation condition through the inverse comparison of modelled substructures and measured transient substructure deflection (see Figure 14 (a)). The method conservatively estimated the long-term (i.e. from 2004 to 2017) formation structural capacity as approximately 60% of its initial design capacity. This therefore indicated that the formation progressively broke down and weakened during the life-cycle. Furthermore, the reliability of the method...
was investigated by applying the calculated EIL boundaries to the measured transient deflections shown in Figure 14 (a), thus comparing the relative local (MDD data) and transient (inverse method) formation strains in Figure 14 (b).

The results showed that the inverse method conservatively estimated the expected relative strains, however, it underestimated the measured SSB and over-estimated the SB Layer strains. This was furthermore attributed to the moulding of the SSB and SB layers resulting in non-monotonic behaviour. As a result, the model did not accurately account for the non-linear formation deterioration and erratic formation response. However, the A and B layers confirm that the inverse model can be utilised for long-term condition assessment of railway formation layers due to the good agreement between the measured and predicted strains within these layers. The method should however be supported with information regarding track formation condition with increase in tonnage to accurately model and evaluate the formation deflections with time, as well as for back-calculation of material properties.
7.4 Application of the Inverse Method

The application of the inverse method was done on train transient substructure deflections measured through remote video monitoring (RVM). This deflection instrument was selected as an economical and practical alternative for deflection measurements at a single measuring point under train loading. The implementation of the method consisted of evaluating the modelled and measured deflection measurements and determining its reliability based on the analysis of the estimated and back-calculated elastic moduli of the formation layers. The application of the inverse method consisted of the following steps:

- Modelling the long-term measured substructure deflections;
- Estimating the model EILs;
- Using the above mentioned information to characterise the actual formation condition in relation to the measured substructure deflections.
Train transient substructure deflections were measured from the top of the formation. Only fully supported sleepers were considered in order to minimize variations in RVM deflection measurements. The inverse model used was adapted to account for the conditions under which the RVM field measurements were obtained namely: 21E Locomotive bogie loading (26 tonnes/axle) and single point measurement at the edge of the sleeper and aligned with the centre line of the sleeper for deflection measurement. Furthermore, the formation layers were modelled according to the long-term formation characterisation, thus the formation elastic moduli were reduced to 60% of that illustrated in Table 3 according to the 2016 structural capacity of the Bloubank test site. For additional information on the structural estimation procedure, refer to Section 7.3

The deflections obtained from the model were directly compared to those obtained from the RVM measurements in Figure 15. As shown, the model accurately replicates the substructure deflection with a difference not exceeding 0.07 mm (B layer boundary). The maximum difference in the distribution of the load in terms of deflection spread is roughly 500 mm. Additionally, Priest et al. (2010) confirmed that linear elastic models are reliable for formation deflection characterisation, however, an inherent difference in curvature is commonly seen due to the fact that granular materials actually behave in an elasto-visco-plastic manner. The displayed discrepancy is therefore negligible. Table 4 shows the estimated relative strain and back-calculated elastic moduli for each individual formation layer with regard to the formation layer boundaries presented in Figure 15.

Table 4 illustrates how the measured formation condition gradually deteriorated with traffic to a state which is similar to that given by the estimated 60% model as confirmed by the A, B and SB elastic moduli. This reduction in formation structural capacity may be predominantly associated with the formation mechanical weathering due to traffic as postulated by Vandoorne et al. (2017). The measured moduli of the mechanically weathered layers differed by no more than 9.1% on average from those estimated by the model. In contrast, the SSB strain hardened with traffic as an increase in elastic modulus was noticed in comparison to the initial and expected values shown in Table 3 and 4 respectively. A previous publication by Vorster & Gräbe (2013) also identified signs of formation strain hardening at the Bloubank test site. The SSB strain hardening shown in Table 4 is, however, assumed to be attributable to a slight conformance error due to the ballast shoulder disturbance that may subsequently affect the adjacent sleepers’ stress distribution, thereby reducing the stress levels and strains experienced within the SSB layer.

The inverse method is thus established as a reliable alternative for the characterisation of formation structural condition based on transient deflection measurements, since good agreement was found for the A, B and SB layers. It is however important to understand that various permutations of substructure properties and characteristics can lead to similar deflection behaviour. Therefore, this method focuses on assessing formation layer condition, based on a predetermined subgrade modulus only. Hence, some predetermined rail track condition information is necessary to model the substructure deflection and accurately estimate the formation structural capacity through the inverse process.
Figure 15: Inverse method on remote video monitoring measurements

Table 4: Comparative analysis of results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strain (%)</td>
<td>Stress (kPa)</td>
</tr>
<tr>
<td>SSB</td>
<td>0.020</td>
<td>73.0</td>
</tr>
<tr>
<td>SB</td>
<td>0.036</td>
<td>51.8</td>
</tr>
<tr>
<td>A</td>
<td>0.060</td>
<td>42.6</td>
</tr>
<tr>
<td>B</td>
<td>0.131</td>
<td>38.2</td>
</tr>
</tbody>
</table>

8 CONCLUSIONS

The following conclusions can be drawn from the results described in this paper:

- The effect of train speed was not an influential factor for formation peak deflection in the case of train speeds less than 80 km/h. The increase in formation peak deflection was directly proportional to the increase in axle loading. Furthermore, neither train speed nor axle load had a well-defined influence on the deflection basin spread.
- Finite element analysis allowed for detailed investigation of railway formation behaviour through different superstructure and substructure compositions as well as for the estimation of railway EILs under simulated train transient loading.
• The commonly assumed 45° influence line was found to be an oversimplification for inverse deflection analysis of railway track formations due to the indirect load transfer and the superposition of bogie wheel loads on the discontinuous track superstructure support.

• Railway formation EILs varied according to the structural capacity of the formation and subgrade layers and can be associated with the transition from shallow to deep substructure behaviour which defines the formation response in relation to the deterioration of the formation layers and the subgrade support.

• The local and transient deflections measured with the MDDs were compared to those calculated by the model. The similarity of the results confirmed the adequacy of the model. The validation of the inverse method was confirmed through the strong agreement between the measured and inversely estimated formation peak strains. The estimated relative formation strains differed by no more than 10.5% on average from the measured peak strains.

• A strong correlation was found between the measured remote video monitoring deflections and the calculated deflections from the long-term inverse method. The method accurately determined the formation structural condition as shown by the measured and estimated (model) formation elastic moduli. The measured moduli of the SB, A and B layers differed by no more than 9.1% on average from those estimated by the model.

In conclusion, railway track structures exhibit complex substructure deflection responses which are primarily affected by the bogie length, axles/bogie, axle loading, sleeper spacing and support as well as the substructure condition. The inherent deflection variability under train transient loading can be minimised for inverse analysis. The railway EILs were characterised and the commonly assumed 45° influence line was determined unsuitable for formation characterisation. The individual formation layers’ deflection and elastic moduli were quantified through the use of the inverse method using transient substructure deflection measurements. Thus, the method was found suitable for estimation of formation condition over its design life.

It is clear that the method can at best provide an estimate of the formation structural condition over time, due to the significant influence of the bogie loading/superstructure interaction, sleeper movement, ballast support and substructure structural capacity on the formation deflection response. These variables can vary significantly over the design period of any ballasted track substructure. It is therefore advised to investigate railway track formations with the support of information on the track substructure composition and condition, as the key to the inverse method is to accurately determine the EIL for the specific substructure conditions. Furthermore, any railway track formation deflection analysis using this mechanistic-empirical method should conform to the following guidelines until further research reinforces the method:

• The inverse analysis of railway track formation should be complemented with practical experience, visual surveys and laboratory testing if required.
• The analysis of elastic moduli of track formation layers should be conducted with the support of track condition information regarding the track design and material specification.
• The results should be evaluated with clear understanding of the approximations and assumptions inherent to the method.

9 ACKNOWLEDGEMENTS

Transnet Freight Rail is gratefully acknowledged for sponsoring the Chair in Railway Engineering in the Department of Civil Engineering at the University of Pretoria. The Transnet Freight Rail staff of the Vryheid and Richards Bay Depots are thanked for granting access and track maintenance assistance.

10 REFERENCES


Vorster, D.J. and Gräbe, P.J., (2013), February. *The Effect of axle load on track and foundation resilient deformation under heavy haul conditions*. In 10th international heavy haul association conference, New Delhi, India.


H.A. Mones Ruiz Martin is a civil engineer, full-time postgraduate researcher of civil engineering at University of Pretoria, with focus on transportation (railway and pavement) engineering. He obtained his B.Eng degree in Civil Engineering, B.Eng (Postgraduate) and M.Eng degree in Transportation Engineering from the University of Pretoria. He also tutors undergraduate classes at University of Pretoria. He is a Candidate Engineer with the Engineering Council of South Africa (ECSA) and a student member of the South African Institution of Civil Engineering (SAICE).

Prof. P.J. Gräbe (Pr Eng, FSAICE), is a civil engineer with experience in the fields of track technology, geotechnology, advanced laboratory testing, field investigations, maintenance models and numerical analysis of track structures. He is Associate Professor: Transnet Freight Rail Chair in Railway Engineering at University of Pretoria, where he lectures under- and post-graduate courses in railway engineering. He is also responsible for railway research, as well as for continuing professional development in the form of short courses presented to industry. He holds a PhD degree from the University of Southampton (UK) and is fellow of the South African Institution of Civil Engineering (SAICE) and registered with the Engineering Council of South Africa (ECSA) as a professional Engineer.
Prof. J.W. Maina (Pr Eng, MSAICE, FSAAE) is a professional pavement engineer, full-time professor of civil engineering at the University of Pretoria (UP). He is an expert in numerous fields of science, engineering and technology (SET) focusing on pavement engineering, which include numerical modelling, characterisation of mechanical properties of materials, technology of contact stresses between tyres and road surfaces, non-destructive field testing, the use of high performance computing and Quality Assurance / Quality Control (QA/QC) in pavement project. Using this expert knowledge, Prof. Maina is now the architect of a suite of software packages catering for different road materials properties, loading characteristics and pavement configuration. He also lectures both undergraduate and postgraduate courses and he also supervises both Masters and Doctoral students at University of Pretoria.